Normal Mode Impedances of a Coupled Pair of Microstrip Transmission Lines

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For guidance in the design of integrated microwave circuit components, data are required on the parameters of symmetrical coupled pairs of microstrip transmission lines. The parameters needed to characterize this structure are the characteristic impedances and velocities of propagation of the two normal modes. In addition, for certain purposes such as investigation of spurious coupling, peak power capability, and gyromagnetic interaction (in the case of nonreciprocal substrate material), information is also required on the r-f field configuration.

From the symmetry of the structure, the normal modes of propagation on a pair of parallel strips of equal width have even and odd symmetry, respectively, with respect to reflection in the central bisecting plane. At sufficiently low frequencies (quasi-static limit) propagation is approximately TEM; hence the impedances of the two modes can be determined from their respective d-c capacitances and low-frequency velocities. The difference in impedances becomes large as the coupling between the strips is increased by reducing the spacing between them. The analogous problem of coupled lines in the case of balanced stripline has been treated by S. B. Cohn.[1] The microstrip problem is complicated by the lower symmetry and the presence of the dielectric-air boundary. Calculations of capacitance, velocity, and impedance of single strips and coupled pairs of strips, using various approximate methods, have been performed recently by H. A. Wheeler,[2] E. G. Cristal,[3] K. C. Wolters and P. L. Clar,[4] G. Policky and H. L. Stover,[5] and others. The physical construction of the coupled microstrip lines is shown in Fig. 1. The arrangement is specified in the conventional way by the parameters W/H and S/H, together with ϵ_s where W is the strip width, H the substrate height, and S the spacing between adjacent edges; ϵ_s, is the dielectric constant of the substrate material. The strip thickness T is taken to be negligibly small.

The method to be described in the present paper was devised with the objective of obtaining an accurate solution of the quasi-static problem, including a rigorous treatment of the inhomogeneous dielectric medium, making efficient use of computer time and obtaining an accurate assessment of the effects of any approximations employed.

Given a harmonic function (potential) which, in the case of a homogeneous medium, satisfies the appropriate Dirichlet boundary conditions at the conducting surfaces, the error which would be incurred by assuming this potential to be the solution of the inhomogeneous-dielectric problem may be expressed as a distribution of "free charge" (surface divergence of a electric displacement) on the air-dielectric boundary. Intuitively, we may imagine placing a compensating distribution of "bound charge" (giving rise to a surface divergence of electric field) on the boundary so as to cancel the free charge. Once found, this bound charge distribution uniquely determines the potential -- hence the fields -- satisfying all conditions of the inhomogeneous-dielectric problem. Clearly, a complication arises from the fact that a compensating charge placed at a point on the boundary makes its own contribution to the divergence of displacement elsewhere on the boundary -- that is, creates additional free charge. Taking this effect into account, the problem takes the form of a Fredholm's integral equation of the second kind, for which the kernel is the electrostatic Green's function corresponding to the conductor configuration of the microstrip transmission line.

Of the various methods available for solution of this type of system, we have utilized an iterative method which lends itself well to machine computation. Our original program was directed toward application of the method to a simplified geometry obtained through a conformal transformation. This procedure gives accurate solutions to the problem of a single strip; we have verified that agreement with the results of Wheeler[2] is excellent. The transformed geometry employed by Wheeler is illustrated in Fig. 2. (The agreement may be viewed as an impressive demonstration of the remarkable accuracy of Wheeler's shrewd approximation.) For the coupled-strip problem, however, we find that there is no advantage in the transformed geometry; in fact, there are substantial disadvantages arising from the intrinsically multiply-connected character of the cross-section. We discovered that the approximations required in order to obtain a workable transformation are, first, seriously damaging to the accuracy of the calculation (particularly in the important range, $W/H < 1$), and second, unnecessary. Unfortunately, at the time of this writing (March 25th) the results of the revised calculations are not quite in presentable form. Our tests of the method have shown it to be both efficient and accurate, however. A full report including data applicable to a wide range of cases of practical importance will be presented at the Symposium. The data include the even- and odd-mode impedances and velocities, in the quasi-static limit, for substrate dielectric constants of 9 and 16 and a wide range of strip widths W/H and spacings S/H. Further details of the potential and fields will be presented, including a discussion of the derived "dielectric Green's function" (Fig. 3) which plays a central role in the calculation.

Wavelength data recently obtained by J. D. Welch and H. T. MacFarland[6] indicate that surface-wave effects cause measurable slowing at frequencies such that the substrate thickness H is greater than about 1% of the characteristic wavelength of the dielectric material. Their data confirm that at lower frequencies the quasi-static approximation described in the present paper is valid, and at higher frequencies it provides useful design guidance.

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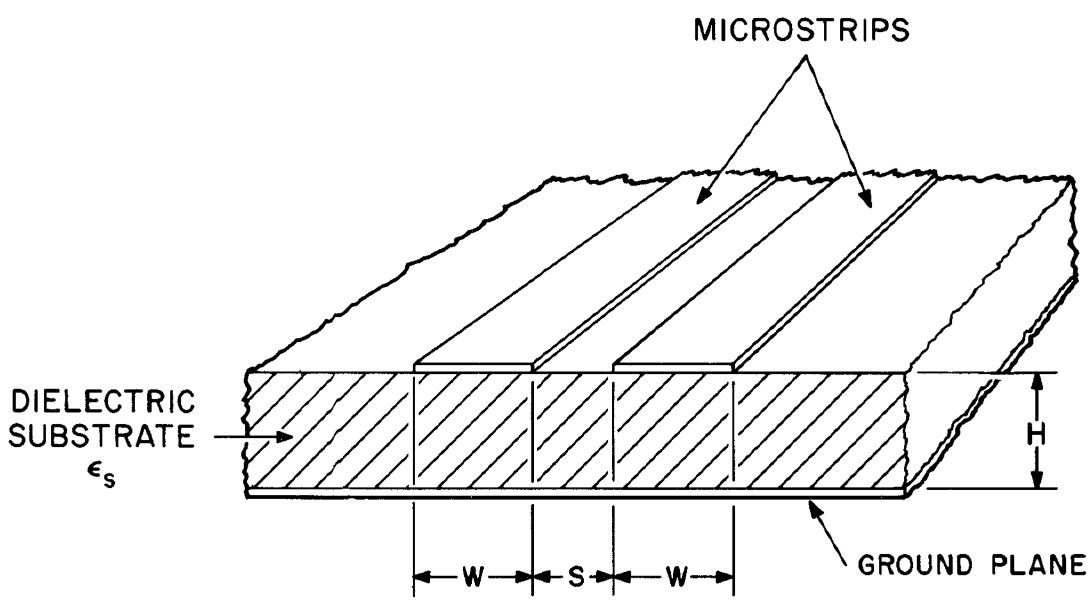

Fig. 1. Coupled microstrip transmission lines.

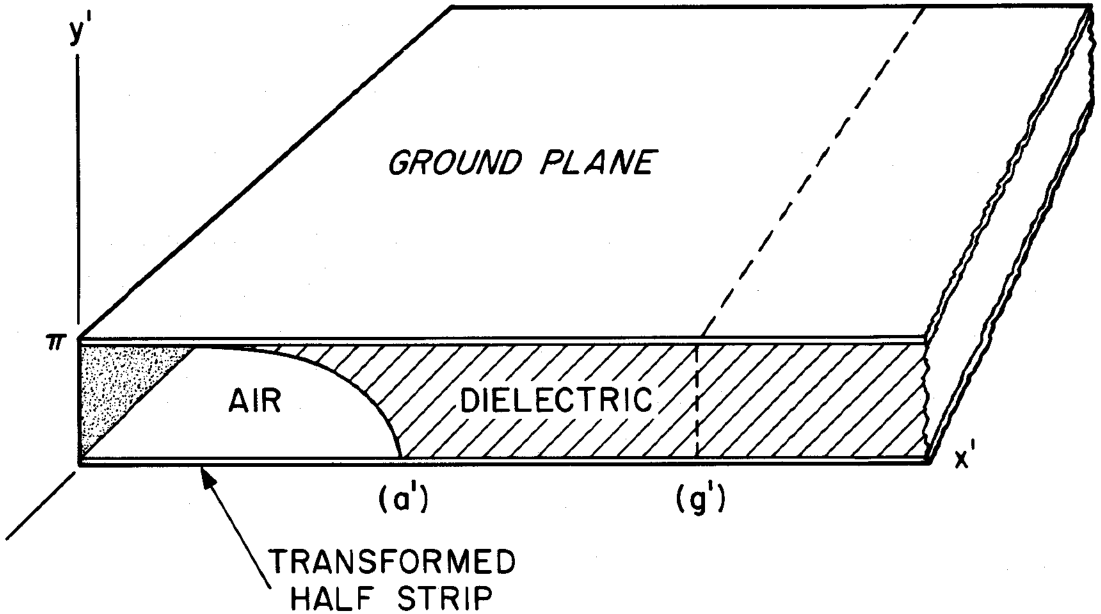

Fig. 2. The conformally-transformed geometry employed by H. A. Wheeler (ref. 2) in the analysis of a single micro-strip transmission line, showing the resulting form of the dielectric-air boundary.

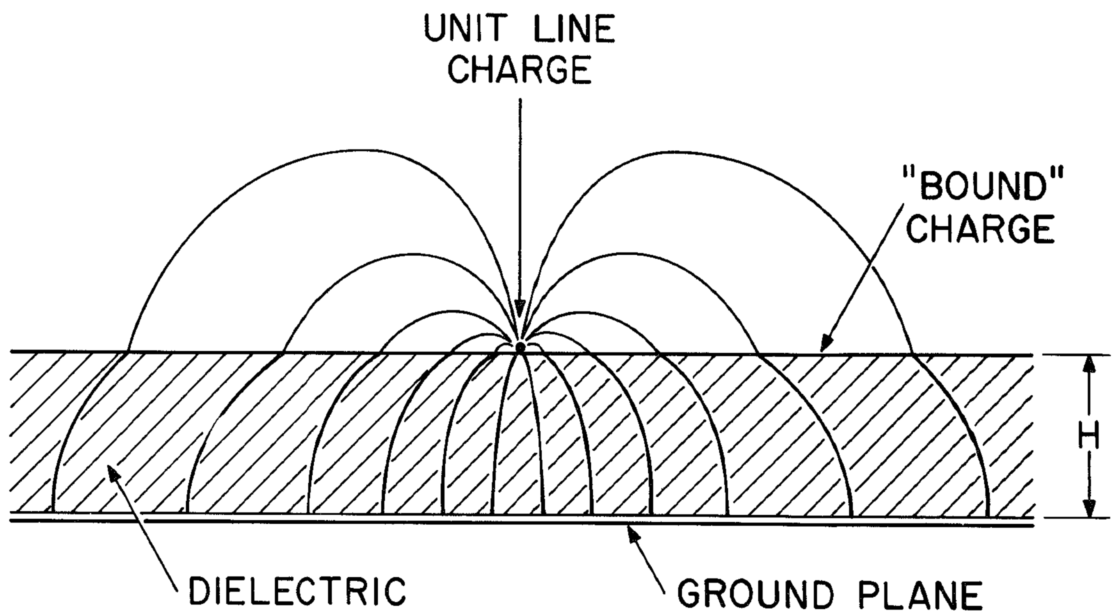

Fig. 3. The "dielectric Green's function" employed as the elementary excitation in the investigation of coupled microstrips, representing a unit charged line together with the "bound charge" which it induces on the dielectric surface.